

QUANTUM ZENO AND QUANTUM ANTI-ZENO EFFECTS

V. Gontis and B. Kaulakys*

Institute of Theoretical Physics and Astronomy, A. Goštauto 12, 2600 Vilnius, Lithuania
**and Vilnius University, Department of Quantum Electronics*

Consequences of the deviation from the linear on time quantum transition probabilities leading to the non-exponential decay law and to the so-called Zeno effect are analysed. Main features of the quantum Zeno and quantum anti-Zeno effects for induced transitions are revealed on simple model systems.

1 Introduction

It is generally accepted that unstable systems decay according to an exponential law which has been experimentally verified on many quantum systems. However, deviation from exponential decay have been predicted for short as well as for very long decay times. The short-time deviation turns out to be very interesting due to its consequences leading to the so-called quantum Zeno effect – the inhibition of decay by repeated observation in very early stages of evolution. Here we discuss shortly the essence of the exponential and non-exponential (nonlinear) decay and effect of frequent measurements on the induced dynamics of simple and multilevel quantum systems.

2 Exponential and non-exponential decay

When the decay probability P_d of the system depends linearly on time, $P_d = \gamma\tau$, for very short time intervals τ soon after preparation of the system, $\gamma\tau \ll 1$, then the survival probability of the system in the initial state is

$$P(t) = 1 - \gamma\tau, \quad \gamma\tau \ll 1.$$

The survival probability as a result of evolution of the system during time $t = n\tau$ is

$$P(t = n\tau) = (1 - \gamma\tau)(1 - \gamma\tau) \cdots (1 - \gamma\tau) = (1 - \gamma\tau)^n. \quad (1)$$

For the large number of short duration evolution intervals, $n \rightarrow \infty, \tau \rightarrow 0, t = n\tau = \text{const}$, from Eq. (1) we have the exponential law

$$P(t) = \left(1 - \frac{\gamma n\tau}{n}\right)^n = \left(1 - \frac{\gamma t}{n}\right)^n = \exp(-\gamma t), (n \rightarrow \infty). \quad (2)$$

Therefore, the linear in time decay process results in the exponential survival probability dependence on the time. Exponential decay is a well known property of the systems whose decay rate is proportional to its undecayed quantity.

On the contrary, if a quantum system undergoes relatively slow, quadratic, transitions to another states soon after preparation or measurement, $P_d = g\tau^2$, then survival in the initial state probability after time $t = n\tau$ of the system subjected to $n - 1$ intermediate measurements at intervals τ is

$$\begin{aligned} P(t = n\tau) &= (1 - g\tau^2)^n = \left(1 - \frac{gn^2\tau^2}{n^2}\right)^n = \left(1 - \frac{(gt^2/n)}{n}\right)^n \\ &\simeq \exp(-gt^2/n) \rightarrow 1, \quad (n \rightarrow \infty, \tau \rightarrow 0, t = n\tau = \text{const}) \end{aligned} \quad (3)$$

Therefore, as a result of such evolution with large number of intermediate measurements the quantum system under consideration remains in the initial state.

In the axiomatics of quantum mechanics it is postulated that any measurement of the quantum system's state projects it onto an eigenstate of the measured observable and causes disappearance of coherence of the system's state. For this reason in the case of quadratic evolution in time of the system soon after preparation or measurement, the repetitive frequent observation of the quantum system can inhibit the decay of unstable [1] system and suppress dynamics of the driven by an external field [2, 3] system. This phenomenon, namely the inhibition or even prevention of the time evolution of the system from an eigenstate of observable into a superposition of eigenstates by repeated frequent measurements during the time of non-exponential dynamics, is called the *quantum Zeno effect* (paradox) [1–4].

Theoretical analysis of deviation from exponential decay has a long history [5]. It is possible to show quite generally from the equations of quantum mechanics that the decay is slower than exponential for both very short and very long times. The proof of such deviations requires only two very general properties of the system (see [5, 6] and references therein): existence of the lower bound to the energy that the decay products can have and the decaying state must have finite energy. In such a case even from the simpler perturbation theory for short evolution time $t \ll \hbar/(E_u - E_l)$ we have

$$\begin{aligned} P_d &= \frac{1}{\hbar^2} \int_{E_l}^{E_u} |V_{EE_0}|^2 \frac{\sin^2 \frac{E-E_0}{2\hbar} t}{\left(\frac{E-E_0}{2\hbar}\right)^2} \rho(E) dE \\ &\simeq \frac{E_u - E_l}{\hbar^2} |V_{EE_0}|^2 \rho(E_0) t^2 \end{aligned} \quad (4)$$

$$P = 1 - P_d = 1 - gt^2 \quad (5)$$

Here E_u and E_l are the upper and lower possible energy of the decaying system, respectively, V_{EE_0} is the matrix element of the interaction potential for transition between the initial state with energy E_0 and the products' state with energy E and $\rho(E)$ is the density of states of the decay products. These results are independent on detailed interaction that causes the decay.

Usually the time-scales of the short-times deviation from the exponential law are very small: about 10^{-23} s for nuclei, 10^{-17} s for atoms (3.6×10^{-15} s for 2P-2S transition of the

hydrogen atom [7]). For such systems, therefore, we have no chance to observe the effect under consideration. However, recently [8] experimental evidence for short-time deviation from exponential decay in quantum tunnelling of ultracold sodium atoms trapped in accelerating optical potential has been presented at the microsecond time-scale.

The quantum Zeno effect, however, is easier tractable and observable not for spontaneous decay to the continuum states but for the induced transitions between discrete states of the quantum system [2, 3, 9] where transition probabilities are nonlinear functions of time for relatively long time intervals. Therefore, further we will analyse only transitions induced by external field.

3 Quantum Zeno effect for induced transitions

Modeling the effect of measurement on the dynamics of quantum system by randomizing the phases of the measured states we may derive equation for the transition probabilities of the two-state system in the resonance field. The simplest time evolution of the two-state wave function $\Psi = a_1 |1\rangle + a_2 |2\rangle$ from time moment $t_k = k\tau$ to $t_{k+1} = (k+1)\tau$ can be represented as

$$\begin{pmatrix} a_1(k+1) \\ a_2(k+1) \end{pmatrix} = \mathbf{A} \begin{pmatrix} a_1(k) \\ a_2(k) \end{pmatrix}, \quad (6)$$

$$\mathbf{A} = \begin{pmatrix} \cos \varphi & i \sin \varphi \\ i \sin \varphi & \cos \varphi \end{pmatrix}, \quad \varphi = \frac{1}{2}\Omega\tau \quad (7)$$

where Ω is the Rabi frequency. Evidently the evolution of the amplitudes from time $t = 0$ till $t = n\tau$ may be expressed as

$$\begin{pmatrix} a_1(n) \\ a_2(n) \end{pmatrix} = \mathbf{A}^n \begin{pmatrix} a_1(0) \\ a_2(0) \end{pmatrix}. \quad (8)$$

One can calculate matrix \mathbf{A}^n by the method of diagonalization of the matrix \mathbf{A} . The result naturally is

$$\mathbf{A}^n = \begin{pmatrix} \cos n\varphi & i \sin n\varphi \\ i \sin n\varphi & \cos n\varphi \end{pmatrix}. \quad (9)$$

For the time interval $T = n\tau = \pi/\Omega$ a certain (with the probability 1) transition between the states takes place.

Measurement of the system's state in the time moment $t = k\tau$ projects the system to the state $|1\rangle$ with the probability $p_1(k) = |a_1(k)|^2$ or to the state $|2\rangle$ with the probability $p_2(k) = |a_2(k)|^2$. After each of the measurement the phases of the amplitudes $a_1(k)$ and $a_2(k)$ are random which results in the absence of the interference terms in the expressions for the probabilities [9]. This results in the equation for the probabilities

$$\begin{pmatrix} p_1(k+1) \\ p_2(k+1) \end{pmatrix} = \mathbf{M} \begin{pmatrix} p_1(k) \\ p_2(k) \end{pmatrix}, \quad (10)$$

where

$$\mathbf{M} = \begin{pmatrix} \cos^2 \varphi & \sin^2 \varphi \\ \sin^2 \varphi & \cos^2 \varphi \end{pmatrix} \quad (11)$$

is the evolution matrix for the probabilities. The evolution from time $t = 0$ until $t = n\tau$ with the $(n - 1)$ intermediate measurement is described by the equation

$$\begin{pmatrix} p_1(n) \\ p_2(n) \end{pmatrix} = \mathbf{M}^n \begin{pmatrix} p_1(0) \\ p_2(0) \end{pmatrix}. \quad (12)$$

Matrix \mathbf{M}^n calculated by the diagonalization method is

$$\mathbf{M}^n = \frac{1}{2} \begin{pmatrix} 1 + \cos^n 2\varphi & 1 - \cos^n 2\varphi \\ 1 - \cos^n 2\varphi & 1 + \cos^n 2\varphi \end{pmatrix}. \quad (13)$$

From Eqs. (12) and (13) we get the quantum Zeno effect [2, 3]: if initially the system is in state $|1\rangle$, the outcome of evolution until the time $T = n\tau = \pi/\Omega$ with the intermediate measurements will be given by the probabilities $p_1(T) = (1 + \cos^n 2\varphi)/2 \rightarrow 1$ and $p_2(T) = (1 - \cos^n 2\varphi)/2 \rightarrow 0$, $(n \rightarrow \infty)$. This result represents the inhibition of the quantum dynamics by measurements and confirms the proposition that act of measurement may be expressed as randomization of the amplitudes' phases.

4 Quantum anti-Zeno effect for dynamics of multilevel systems

Consider now the measurement effect on quantum dynamics of sufficiently more complex systems, i.e. on the induced transitions between states of multilevel system with the quantum suppression of chaotic dynamics. It is natural to expect that frequent measurements of suppressed system will result in additional inhibition of its dynamics.

In general the Schrödinger equation for strongly driven multilevel systems can not be solved analytically. However, the mapping form of quantum equations of motion greatly facilitates investigation of stochasticity and quantum – classical correspondence for chaotic dynamics. From the standpoint of an understanding of manifestation of the measurements for the dynamics of the multilevel systems the region of large quantum numbers is of greatest interest. The simplest system in which the dynamical chaos and quantum localization of states may be observed is a system with one degree of freedom described by the nonlinear Hamiltonian $H_0(I)$ and driven by the periodic $V(\theta, t) = k \cos \theta \sum_j \delta(t - j\tau)$ kicks [10, 11]. Here the convenient variables, angle θ and action I , are introduced. Integrating the Schrödinger equation over the period τ we obtain a map [11]

$$a_m(t_{j+1}) = e^{-i\beta_m} \sum_n a_n(t_j) J_{m-n}(k), \quad \beta_m = H_0(m)\tau, \quad t_j = j\tau \quad (14)$$

for the amplitudes $a_m(t_j)$ before the appropriate j -kick in expansion of the state function $\Psi(\theta, t)$ through the eigenfunctions, $\varphi_m = i^{-m} e^{im\theta} / \sqrt{2\pi}$, of the action $I = -i \frac{\partial}{\partial \theta}$. Here $J_m(k)$ is the Bessel function and the phase factor i^{-m} is introduced for maximal simplification of the map.

Quantum dynamics represented by map (14) with the non-linear Hamiltonian $H_0(I)$ is similar to the classical one only for some finite time $t < t^* \simeq \tau k^2/2$, after which it reveals an

essential decrease of the diffusion rate asymptotically resulting in the exponential localization of the system's state with the localization length $\lambda \sim k^2/2$ [10, 11].

Each measurement of the system's state between $(j - 1)$ and j kicks projects it onto one of the state φ_m with the probability $P_m(t_j) = |a_m(t_j)|^2$. After such a measurement the phase of the amplitude $a_m(t_j)$ is random. Therefore, the influence of the measurements for further dynamics of the system may be expressed as replacement of the amplitudes $a_m(t_j)$ by the amplitudes $\exp[i2\pi g_m(t_j)] a_m(t_j)$, where $g_m(t_j)$ is a random number in case of measurement of the φ_m -state's population before the j kick and equals zero in absence of such a measurement. So, we may analyze the influence on the dynamics of measurements performed after every kick, after every N kicks or of the measurements just of some states, e.g. only of the initial state, and observe the reduction of the quantum localization effect in a degree depending on the extent and frequency of the measurement [9]. In the case of measurement of all states after every kick we have the uncorrelated transitions between the states and diffusion-like motion with the quantum diffusion coefficient in the n -space

$$B(n) = \frac{1}{2\tau} \sum_m (m - n)^2 J_{m-n}^2(k) = \frac{k^2}{4\tau} \quad (15)$$

which coincides with the classical one. Therefore, the quantum evolution of frequently observable chaotic system is more classical-like than dynamics of the isolated system (see also [12]). On the other hand, the repetitive measurement of the multilevel systems with quantum suppression of classical chaos results in delocalization of the states superposition and acceleration of the chaotic dynamics which is opposite to the quantum Zeno effect in simple few-level systems. Since this effect is the reverse of the quantum Zeno effect we have called this phenomenon the *quantum anti-Zeno effect* [9].

5 Conclusion

The essence and consequences of the exponential and non-exponential (nonlinear) decay and the effect of repetitive measurement on quantum dynamics of driven by an intensive external force of simple few-level systems as well as of multilevel systems that exhibit the quantum localization of classical chaos has been discussed. Frequent measurement of the simple system yields to the quantum Zeno effect – prevention of time evolution, while that of the suppressed multilevel quantum system, which classical counterpart exhibits chaos, results in the delocalization of the quantum suppression. This outcome is the opposite to the quantum Zeno effect. Therefore, we may call this phenomenon the *quantum anti-Zeno effect*. Furthermore, in the limit of the frequent full measurement or unpredictable interaction with the environment the quantum dynamics of multilevel quasiclassical systems approaches the classical motion.

References

1. **B. Misra and E. C. G. Sudarshan**, J. Math. Phys., V. 18, p. 756 (1976); **C. B. Chiu, E. C. G. Sudarshan, and B. Misra**, Phys. Rev. D, V. 16, p. 520 (1977).

2. **R. J. Cook**, Phys. Scr., V. T21, p. 49 (1988).
3. **W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland**, Phys. Rev. A, V. 41, p. 2295 (1990).
4. **P. Knight**, Nature, V. 344, p. 493 (1990); **A. Schenzle**, Contemp. Phys., V. 37, p. 303 (1996).
5. **L. A. Khalfin**, Zh. Eksp. Teor. Fiz., V. 33, p. 1371 (1958) [Sov. Phys. JETP, V. 6, p. 1503 (1958)]; **J. Swinger**, Ann. Phys., V. 9, p. 169 (1960); **L. Fonda, G. C. Ghirardi, and A. Rimini**, Rep. Prog. Phys., V. 41, p. 587 (1978); **G.-C. Cho, H. Kasari, and Y. Yamaguchi**, Prog. Theor. Phys., V. V. 90, p. 803 (1993).
6. **P. T. Greenland**, Nature, V. 387, p. 548 (1997).
7. **P. Facchi and S. Pascazio**, Temporal behavior and quantum Zeno region of an excited state of the hydrogen atom, *to be published*.
8. **S. R. Wilkinson et al.**, Nature, V. 387, p. 575 (1997).
9. **B. Kaulakys and V. Gontis**, Phys. Rev. A, V. 56, p. 1131 (1997); e-print archives <http://xxx.lanl.gov/quant-ph/9708024>.
10. **G. Casati, B. V. Chirikov, D. L. Shepelyansky, and I. Guarneri**, Phys. Rep., V. 154, p. 77 (1987); **F. M. Izrailev**, Phys. Rep., V. 196, p. 299 (1990).
11. **V. G. Gontis and B. P. Kaulakys**, Liet. Fiz. Rink., V. 28, p. 671 (1988) [Sov. Phys.-Collec., V. 28(6), p. 1 (1988)].
12. **B. Kaulakys**, Lithuanian J. Phys., V. 36, 343 (1996); e-print archives <http://xxx.lanl.gov/quant-ph/9610041>.